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BCS-012

BACHELOR OF COMPUTER APPLICATIONS (BCA) (Revised)

Term-End Examination

09411

June, 2017

BCS-012 : BASIC MATHEMATICS

Time : 3 hours *Maximum Marks : 100*

Note : *Question number 1 is* ***compulsory****. Attempt any* ***three*** *questions from the remaining four questions.*

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1. (a) Show that

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (b - a)(c - a)(c - b). \qquad 5$$

(b) Using determinants, find the area of the triangle whose vertices are (1, 2), (– 2, 3) and (– 3, – 4). *5*

(c) Use the principle of mathematical induction to prove that

$$\frac{1}{(1)(2)} + \frac{1}{(2)(3)} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

for every natural number n. *5*

(d) If the first term of an A.P. is 22, the common difference is -4, and the sum to n terms is 64, find n. 5

(e) Find the points of discontinuity of the following function : 5

$$f(x) = \begin{cases} x^2, & \text{if } x > 0 \\ x + 3, & \text{if } x \leq 0 \end{cases}.$$

(f) If $y = ax + \frac{b}{x}$, show that

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 0.$$ 5

(g) Prove that the three medians of a triangle meet at a point called centroid of the triangle which divides each of the medians in the ratio 2 : 1. 5

(h) Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$. 5

2. (a) Solve the following system of linear equations using Cramer's rule : 5

$$x + y = 0, \quad y + z = 1, \quad z + x = 3$$

(b) If $A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, find a and b. 5

(c) Reduce the matrix

$$A = \begin{bmatrix} 5 & 3 & 8 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

to normal form and hence find its rank. 5

(d) Show that n(n + 1) (2n + 1) is a multiple of 6 for every natural number n. 5

3. (a) Find the sum of an infinite G.P. whose first term is 28 and fourth term is $\frac{4}{49}$. 5

(b) Use De Moivre's theorem to find $(\sqrt{3} + i)^3$. 5

(c) If 1, ω, ω^2 are cube roots of unity, show that

$$(2 - \omega)(2 - \omega^2)(2 - \omega^{10})(2 - \omega^{11}) = 49.$$ 5

(d) Solve the equation

$$2x^3 - 15x^2 + 37x - 30 = 0,$$

given that the roots of the equation are in A.P. 5

4. (a) A young child is flying a kite which is at a height of 50 m. The wind is carrying the kite horizontally away from the child at a speed of 6·5 m/s. How fast must the kite string be let out when the string is 130 m ? 5

(b) Using first derivative test, find the local maxima and minima of the function

$$f(x) = x^3 - 12x.$$ *5*

(c) Evaluate the integral

$$I = \int \frac{x^2}{(x+1)^3} dx.$$ *5*

(d) Find the length of the curve

$$y = 3 + \frac{1}{2}(x) \text{ from } (0, 3) \text{ to } (2, 4).$$ *5*

5. (a) If $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then prove that $\vec{a} + \vec{b}, \vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are also coplanar. *5*

(b) Find the Vector and Cartesian equations of the line passing through the points (– 2, 0, 3) and (3, 5, – 2). *5*

(c) Best Gift Packs company manufactures two types of gift packs, type A and type B. Type A requires 5 minutes each for cutting and 10 minutes each for assembling it. Type B requires 8 minutes each for cutting and 8 minutes each for assembling. There are at most 200 minutes available for cutting and at most 4 hours available for assembling. The profit is ₹ 50 each for type A and ₹ 25 each for type B. How many gift packs of each type should the company manufacture in order to maximise the profit ? *10*